Optimization of maintenance actions in train operating companies

Fertagus case study

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Abstract

Nowadays, efficient transport throughout Europe and the world has become a prerequisite for both freight and passenger travels. Railway transport still has to improve in EU in order to win market share from roads and sea in the future, but it is already an important mean of transport all over Europe. In Lisbon metropolitan area, Fertagus was the first private train operating company. This train operating company is running a line between Roma-Areeiro and Setubal and has its own maintenance yard. Therefore, optimizing maintenance costs is one of the main objectives of Fertagus train operating company. This work presents a mathematical model (which is a mixed integer linear programming model) that was implemented in FICO Xpress software. The model was validated and illustrated with a small-scale example. This mathematical model gives optimal technical planning as an output which reduces the cost of preventive maintenance. Real data was collected during meetings at Fertagus maintenance yard and is used in this work to obtain the minimal costs possible for preventive maintenance. Some sensitivity analysis is performed on some parameters of the mathematical model.

Resumo

Atualmente, o transporte eficiente em toda a Europa e no mundo tornou-se-se um pré-requisito tanto para viagens de passageiros como de mercadorias. O transporte ferroviário ainda tem de melhorar na UE, a fim de ganhar quotas de mercado ao transporte rodoviário e marítimo no futuro, mas já é um meio importante de transporte em toda a Europa. Na área metropolitana de Lisboa, a Fertagus foi a primeira companhia ferroviária privada. Esta empresa ferroviária opera uma linha entre Roma-Areeiro e Setúbal e tem as suas próprias oficinas de manutenção. Assim, otimizar os custos de manutenção é um dos principais objetivos da Fertagus. Este trabalho apresenta um modelo matemático (que é um modelo de programação linear inteira mista) que foi implementado no software FICO Xpress. O modelo foi validado e ilustrado com um exemplo de pequena escala. Este modelo matemático oferece um melhor planeamento como resultado e reduz o custo da manutenção preventiva. Dados reais foram recolhidos durante as reuniões nas oficinas de manutenção da Fertagus e são utilizados neste trabalho para obter os custos mínimos possíveis para a manutenção preventiva. Algumas análises de sensibilidade são realizadas em alguns parâmetros do modelo matemático.

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u	train unit
t	time unit
i	maintenance activity
p	spare part
l	maintenance line
U	set of train units u
I	set of maintenance activities i
T	set of time units t
P	set of spare parts p
L_i	set of available maintenance lines l in the maintenance yard for maintenance activity i
MA_cost_i	cost of maintenance activity i
T_i	period of maintenance activity i (in time unit)
Δ_i	amount of work required to perform maintenance activity i (in man-hour)
$duration_i$	duration of the maintenance activity i. (Note; this calculated as the ratio between Δ_i and the number of men needed to perform the maintenance activity i)
SP_cost_p	cost of having a spare part p per time unit t
κ_{ip}	number of spare parts p needed to perform maintenance activity i
R_p	duration of the maintenance of spare part p (in time unit)
A_p	maximum amount of spare parts p
O_{ui}	time interval between last maintenance activity i and beginning of planning horizon for train unit u
H	planning horizon
S	shunting cost
k	maximum working load per time unit t (in man-hours)
max_time	maximum working time per time unit t (in hours)
N	number of maintenance activities i (Note: it is the cardinality of the set I)
delay	amount of time needed to move a train from a maintenance line l (in hours)
u_1	maximum number of train units available
u_2	number of train units needed to perform daily service
x_{uitl}	binary variable set to 1 if maintenance activity i is performed on train unit u at time unit t, in maintenance line l; and set to 0 otherwise.
y_{ut}	binary variable set to 1 if train unit u is under maintenance at t time unit; and set to 0 otherwise.
U_p	non-negative integer variable corresponding to the minimum amount of spare part required to perform the technical planning

Chapter 1 Introduction

This first chapter introduces the research topic of the present dissertation, providing a brief background of both European and Lisbon metropolitan area railway systems, as well as an overview of maintenance planning. After a subsection on the context of the study, research objectives, methodology and the structure of the dissertation are presented.

1.1 Context

1.1.1 Railway mobility in Europe

Within the borders of western countries of EU, the railway lines connect correctly the majority of the regions to one another. The railway quality inside borders is due to railway being a traditional transport in most of the western countries of Europe. However, an important issue concerning its development is the way people see this industry nowadays. *"From a customer perspective, the quality of rail services continues to be perceived as insufficient. Only 58% of Europeans are satisfied with their rail services, while just 51% are satisfied with railway stations in Europe. Rail still does not come across as a user-friendly transport mode, with 19% of Europeans simply not taking the train because of accessibility issues."* (EC 2011). This will have to be improved if the railway industry wants to enter new markets in the future, such as freight transport which is now mainly done through roads and sea.

Indeed, if the railway industry is efficient nationwide in a large part of EU countries, when it comes to longer journeys across Europe, railway transport has been overtaken by planes, roads and boats for both freight and passenger transport. It would be advisable not to rely mainly on these means of transportation because of the petroleum dependency of EU. In 2010, the oil import bill was around EUR 210 billion for the EU. This is why, according to the White paper on transport, the goal is to be able to, "*triple the length of the existing high-speed rail network by 2030 and maintain a dense railway network in all Member States. By 2050 the majority of medium-distance passenger transport should go by rail*." (EC 2011). Of course, one of the main challenges is to be able to connect all European lines to one another, guarantee availability and reduce delays to a minimum. Even if the situation has improved greatly over the last years, some improvements are still needed if railway transport want to take over market shares from road transport.

Most of the EU lines used to have specific national infrastructure, electrification or control-command systems which means that connecting them to one another had been an issue in the past years. Moreover, the lines connecting borders were often not built yet which implied some additional costs. This is why, in order to be able to achieve EU objectives, an organisation, the CEF (Connecting Europe Facility) has been created to supervise this transition. But the European Commission not only asked to create a wider railway network; "by 2030, the goal for transport will be to reduce GHG emissions to around 20% below their 2008 level." (EC 2011). In order to achieve this ambitious goal, both railway companies and train manufactuers will have to do some serious research. The project Horizon 2020 that will run until 2020, is expected to have a cost of "EUR 77 billion, of which roughly EUR 6.339 million will go towards support to smart, green and integrated transport." (EC 2011). Railway innovation will get EUR 450 million from EU and just as much from railway industries taking part in Shift2Rail program. This will enable research to be done on in-need fields. Furthermore, interoperability will be required within EU borders, as the one that had been achieved for the current train fleet. Indeed, if one country chooses to go for an electricity powered solution and another for a biofuel energy solution; the two systems will not be compatible in the brand new sustainable European network.

As all this research will have a cost for railways companies, they will have to find ways to redistribute their budget in order to be able to bear the transition to renewable transport solutions. One way would be to optimize the maintenance cost in order to invest the savings in innovation and development of the future of railway transport in EU. The present work lines up into the EU vision, even if it is only applied to a train operating company's maintenance yard in Portugal at Coina.

1.1.2 Railway system in Lisbon metropolitan area

In the metropolitan area of Lisbon, the railway system works the same way as in most EU capitals; where several stations connect the suburbs to the city center. In order to be able to manage the transport flows, two distinct train operating companies run five lines (Table 1). These companies are ***CP*** (Comboios de Portugal) which is a state-owned company and ***Fertagus*** which is a private company. It is interesting to underline that railway operators got separated from companies managing railway infrastructures ten years after the European Railway Directive 91/440/EC in 1991. It was obviously not effective immediately but now, all lines within Lisbon Metropolitan area are managed by IP (Infraestructuras de Portugal) and operated by CP and Fertagus. In the previous years, the only railway operator was CP which might explain that Fertagus is only operating one line in Lisbon urban area, whereas CP is also operating regional and inter-regional trains.

Table 1: Railway lines within Lisbon metropolitan area

Line	Route	Operator
Azambuja	From: *Azambuja* and *Castanheira do Ribatejo* To: *Santa Apolonia* and *Alcântara-Terra*	CP
Cascais	From: *Cascais* To: *Cais do Sodré*	CP
Sado	From: *Praias do Sado* To: *Barreiro*	CP
Sintra	From: *Sintra* and *Mira Sintra-Meleças* To: *Alverca, Oriente* and *Rossio*	CP
Setubal	From: *Roma-Areeiro* To: *Setubal*	Fertagus

A map of the railway network of Lisbon metropolitan area can be found in Figure 1, transport above water is represented even through the river is not shown on this map. Fertagus' line is pictured in dark blue from Roma-Areeiro to Setubal.

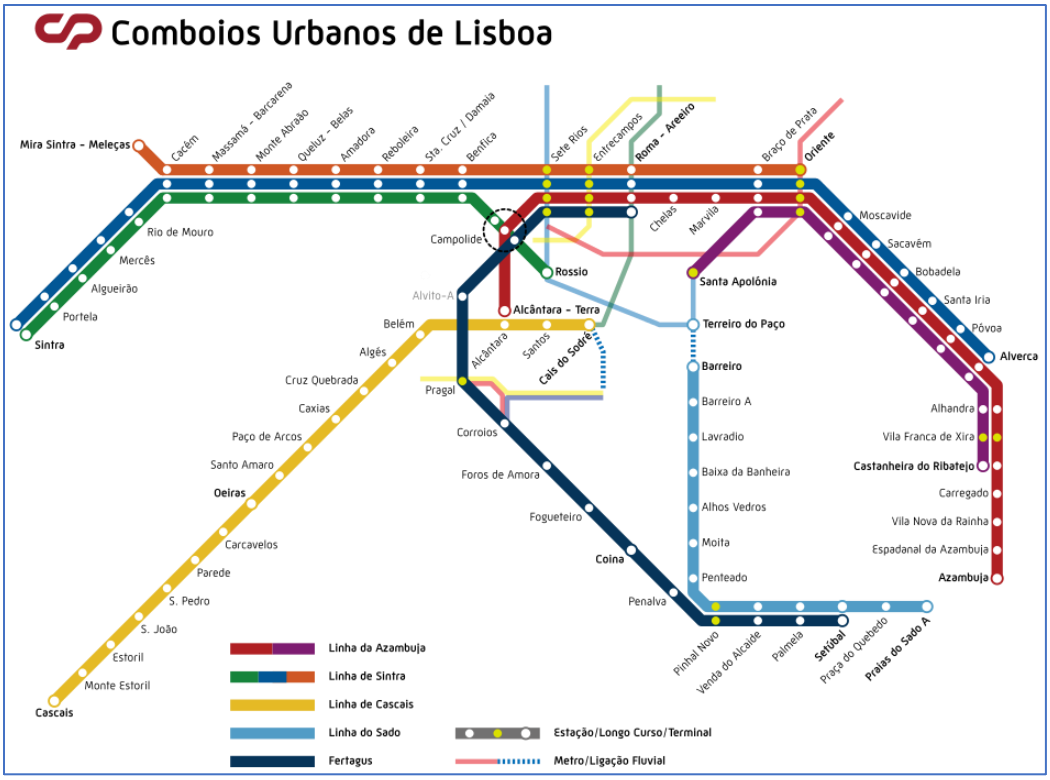

Figure 1 : Railway's map of Lisbon metropolitan area

1.1.3 Maintenance planning in transportation companies

In transportation companies, maintenance has a critical impact on both safety and availability. Indeed, it can be understood that if a vehicle is not maintained at all, components would fail more and more throughout use. This is of course not advisable as it would reduce availability of the fleet and could lead to critical safety issues. Therefore, companies have understood that even if maintenance costs could be quite high, it would guarantee fleet's availability and would prevent accidents. Both of these factors have a major impact on the corporate's image which is something that should be taken into consideration. In order to ensure safety, vehicles manufactuers require that preventive maintenance is performed within deadlines. In general, maintenance is required at a given mileage or a given time interval for trains, planes, buses but it is also mandatory for private cars.

When it comes to maintenance, there are two ways to proceed: it can either be done when the maintenance deadline is reached or when a failure occurs. The first kind of maintenance is called ***preventive maintenance***, while the second is named ***corrective maintenance***.

According to Do *et al.* (2015), “it is assumed that after a preventive action, the maintained component becomes as good as new [whereas] that a corrective action restores the component involved into a state as bad as old.” So that safety is ensured, preventive maintenance interval must be optimized in order to keep the failure rate of the vehicle components under a satisfactory level. Of course, it could be tempting to perform preventive maintenance at very small intervals in order to have very low failure rates. However, if unnecessary preventive actions are performed too often, maintenance costs would dramatically increase and, moreover, early maintenance can sometimes trigger component failure. Thereby, preventive maintenance intervals should be chosen wisely by taking into account these two factors. As a result, preventive maintenance can then be optimized in order to get the cheapest maintenance costs possible that still fulfil every deadline of the company vehicles. Corrective maintenance on the other hand has a random nature and is hard to predict and thus hard to optimize. In chapter 2, most of the papers refer to preventive maintenance.

1.2 Research Objectives and methodology

The aim of this dissertation is to minimize the cost spent on maintenance by train operating companies, more precisely the objective is to create a technical maintenance planning that minimizes the total maintenance cost. The maintenance model developedis then adapted to the Fertagus case study.

In order to achieve this goal, the following steps were pursued:

- Literature review on maintenance planning;
- Adaptation of an optimization model described in the literature review to the case study;
- Data collection and model implementation in a mixed-integer linear programming model;
- Results analysis and discussion;
- Conclusions, limitations, and identification of future research.

The first step of this work was an in-depth state of the art on maintenance planning and associated optimization. In this dissertation only more specific papers will be summarized as the more general references were not considered to be relevant to this study.

By studying numerous models, Doganay and Bohlin (2010) optimization model and its latest version by Bohlin and Warja (2015) were selected as the most suitable ones. It was found that their model could be easily adapted to fit the Fertagus case study. Some adjustments were needed and are actually a research contribution of the present work. Among the modifications performed, additional technical constraints associated with the maintenance yard configuration were added.

In order to be able to adjust the initial model, meetings with the maintenance director and the maintenance supervisor were organized. The objective of these meetings was to become more familiar with the technical constraints within the maintenance yard of Fertagus and collect the relevant data.

1.3 Structure

This dissertation is divided into:

1. Introduction – This first chapter introduces the research topic of the present dissertation, providing a brief background on the topic. Then, it briefly presents both European and Lisbon metropolitan area railway systems, as well as an overview of maintenance planning.

2. State of the art in maintenance planning – In chapter 2, the most appropriate papers studied during this dissertation are presented and summarized in chronological order. A distinction was made between papers which focus on maintenance optimization planning in general and those applied directly to transportation fields.

3. A mixed integer linear programming model – In chapter 3, the optimization model is described and both the objective function and the constraints are explained in detail. Although the initial model by Doganay and Bohlin (2010) on the maintenance optimization for train fleet was considered appropriate, the final version of this model was changed and adapted to better fit Fertagus case study.

4. Model implementation and validation – In chapter 4, an illustrative example and its implementation is explained in detail in order to provide a better understanding of the mathematical model. In a first subsection, the implementation in FICO Xpress is described. In the second subsection, the parameters of the mathematical model are displayed and in a third subsection the results of the mathematical model are explained.

5. The case study of Fertagus operating company – In chapter 5, Fertagus train operating company is presented briefly and problem specifications are introduced. The parameters of the mathematical model are displayed in a table with all values given in monetary units for the sake of confidentiality.

6. Results and Discussion – In chapter 6 several analyses regarding the optimization model are presented. Firstly, an analysis of optimality gap over calculation time; the optimality gap mentioned above, is calculated as the percentage of the ratio of the difference between the value of the objective function and the lower bound and the value of the objective function. In the next subsections, a sensitivity analysis is conducted on both the shunting cost component and the maximal working time per week. It should be made clear that the shunting cost is the cost related to moving trains to the maintenance yard.

7. Conclusions and future research – In the final chapter, conclusions of the performed research can be found as well as the discussion of potential limitations and possible further steps of future research.

Chapter 2 State of the art in maintenance planning

The most appropriate papers studied during this dissertation are presented and summarized below. A distinction has been made between papers which focus on maintenance planning optimization in general and those applied directly to transports.

2.1 Maintenance planning in general

Almgren *et al.* (2009) highlighted that components have either deterministic or stochastic lives. When the failure of a component induces the failure of the system, it is known to have a deterministic life. If it is not the case, then it has a stochastic life. The goal of the paper is to predict when replacement of the equipment is needed so that the cost of the maintenance would be minimum. Indeed, it has been shown that a maintenance which is performed too often could trigger some components failure.

Vu *et al.* (2014) studied the effect of grouping preventive maintenance on multi-components systems. In order to do this study, the authors made a distinction between two types of components. A component can be either critical, if the failure of this component induces the shunt down of the system, or non-critical if the failure does not induce the shunt down of the system. It must be said that the shunt down of one component may influence the lifetime of other components, but this effect is hard to predict. In order to perform preventive maintenance, one has to define sets of components inside the system. The so-called "minimal cut sets" are chosen such that they contain the smallest number of components sufficient to cause the system failure. A critical group would contain at least one critical component.

Do *et al.* (2015) also optimized maintenance using a dynamic maintenance decision rule over a rolling horizon. In order to do so, it is suggested to perform grouping preventive maintenance on systems whose components are connected in series. However, one must keep in mind that performing grouping of preventive maintenance can also increase the cost if the preventive maintenance is performed too early or too late. A major issue of this research is then to find the right maintenance groups; such as maintenance could be performed during system's breaks under a limited number of repairmen and would still be cost saving. For both components in series and multi-components system, dynamic grouping is done in four steps: i) maintenance optimization at component level; ii) tentative planning; iii) grouping optimization; iv) updating. In order to be able to give an optimized maintenance planning which respects both the availability constraint and a fixed number of repairmen, two algorithms are implemented: MULTIFIT and genetic algorithm (GA).

2.2 Maintenance planning in transportation companies

Maintenance optimization in transportation companies has been studied a lot over the last years, and some papers were particularly useful to conduct the present study on Fertagus train set. A state of the art, presented in chronological order, can be found below.

Haghani and Shafahi (2002) studied a way to perform buses' maintenance mostly during their idle time in order to reduce the number of maintenance hours for vehicles that are pulled out of their service for inspection. The solution of the optimization program is a maintenance schedule for each bus due for inspection as well as the minimum number of maintenance lines that should be allocated for each type of inspection over the scheduled period.

Maróti and Kroon (2007) focused on finding a way to allocate to daily service a train that is due for maintenance in a maintenance yard far from the train current location. The objective is to maximize the journeys with passenger on board for a train that is due for upkeep. Indeed, if the train goes as an empty train to the next maintenance, it would significantly increase the cost of the maintenance activity. In order to solve this problem, the authors suggest using an interchange model which modifies the current plan by replacing the regular transitions by combinations of interchanges between the former tasks. Of course, the point is to lead each urgent train unit to maintenance within the deadline.

Technical planning has been studied by Doganay and Bohlin (2010) and their model has been extended by Bohlin and Wärja (2015). In this kind of planning, the time unit is a week as it is not relevant to have a detailed schedule on more than two weeks ahead of the current date. However, knowing how many trains would be maintained on a given week is valuable information. Indeed, it is useful to verify that not too many trains are under maintenance on a given week or that enough spare parts are available to perform the task. It was shown that taking spare parts into account leads to better cost savings, as it removes conflicts caused by too many trains requiring the same spare part at the same time. In Bohlin and Wärja work, inclusions in the maintenance tasks are added, i.e. if a task is included in another, there is no need to perform both in a row.

Bazargan (2015) studied how to minimize the cost of maintenance and maximize aircraft availability and then compared with several possible planning: closest to maintenance; furthest to maintenance; random maintenance; cheapest next maintenance; equal aircraft utilization. This was a study for a flight training school, and it is interesting to notice that they selected the planning with the smallest number of maintenance activities even if it was not the cheapest one. It was interesting to realize that companies would often select user-friendly solutions over solutions harder to implement even if they are more optimized.

Lai, Wang, and Huang (2017) improved the efficiency of rolling stock usage and automate the planning process. The planners are currently doing it manually with a horizon of two days which can lead to myopic decisions, far from the optimum plan. The main target of the objective function is to minimize the gap between the current mileage of the train set and the upper limit each day in order to find the best planning possible.

2.3 Summary of maintenance optimization papers

Table 2 provides a summary of the above-mentioned papers on maintenance optimization. It also gives details on the optimality criterion and on the optimal function that were chosen by the authors.

Table 2 : Summary of maintenance optimization papers

Author(s)	**Goal/Focus/Contribution**	**Optimality criterion**	**Optimal function**
Vu *et al.* (2014)	Maintenance groups	Maintenance cost	Maintenance activity
Do *et al.* (2015)	Maintenance groups	Maintenance cost	Maintenance activity
Almgren *et al. (2009)*	Reliability	Maintenance cost	Replacement of the equipment
Doganay and Bohlin (2010)	Working force and preventive maintenance intervals	Maintenance cost	Maintenance activity and spare parts required
Bohlin and Wärja (2015)	Working force and preventive maintenance intervals	Maintenance cost	Maintenance activity and spare parts required
Haghani and Shafahi (2002)	Preventive maintenance intervals and maintenance yard limitations	Amount of time where buses are pulled out of service for inspection	Maintenance activity
Bazargan (2015)	Preventive maintenance intervals	Availability	Maintenance activity
Maróti and Kroon (2007)	Interchange model	Empty trains going to maintenance yard	Maintenance activity
Lai et al. (2017)	Maintenance planning	Gap between the current mileage of the train set and the upper limit each day	Maintenance activity

Chapter 3 A Mixed-Integer Linear Programming model

In chapter 3, the optimization model is described and both the objective function and the constraints are explained in detail. Although the initial model by Doganay and Bohlin (2010) on the maintenance optimization for train fleet was considered appropriate, the final version of this model was changed and adapted to better fit Fertagus case study. After the presentation of the mathematical model, the implementation on FICO Xpress optimization software is displayed.

3.1 A mixed integer linear programming model

Railway companies were required to reduce their gas emission by 20% by 2030; but the research induced by this obligation will have significant costs that companies must bear. One way could be optimizing the cost of maintenance in order to be able to redistribute the savings to areas in need. In order to achieve this goal, the program created suggests a technical planning with the smallest maintenance cost possible. Indeed, Doganay and Bohlin (2010) showed that it was often not useful to have a detailed planning more than two weeks ahead of the current date. However, it is often very handy to have in advance a less detailed maintenance planning. It enables the maintenance manager to spot any major issues that could happen before they actually occur. A technical planning could reveal that on a given week too many maintenance activities are scheduled considering the number of working men available.

The mathematical model was adapted from both Doganay and Bohlin (2010) and Bohlin and Wärja (2015). As a consequence, all cost components of the objective function are from the first paper; except for the penalty cost which is from the 2015 paper. Some adjustments were also needed and are actually a research contribution of the present work. Additional technical constraints associated with the maintenance yard configuration were added; as well as the allocation of maintenance activity to one specific line of the maintenance yard depending on the requirement of the maintenance activity.

This optimization model uses three types of variables to optimize preventive maintenance costs. The first one is to indicate if during a given week, a maintenance task should be performed on a specific train. The second one is to say if during a given week, a specific train is shunted. Finally, the third type of variable is to indicate the optimal number of each different kind of spare part.

The next subsection details the indices, sets, parameters, constants, decision variables and the objective function.

3.2 Indices

u	train unit
t	time unit
i	maintenance activity
p	spare part
l	maintenance line

3.3 Sets

U	set of train units u
I	set of maintenance activities i
T	set of time units t
P	set of spare parts p
L_i	set of available maintenance lines l in the maintenance yard for maintenance activity i

3.4 Parameters

MA_cost_i	cost of maintenance activity i
T_i	period of maintenance activity i (in time unit)
Δ_i	amount of work required to perform maintenance activity i (in man-hour)
$duration_i$	duration of the maintenance activity i. (Note; this calculated as the ratio between Δ_i and the number of men needed to perform the maintenance activity i)
SP_cost_p	cost of having a spare part p per time unit t
κ_{ip}	number of spare parts p needed to perform maintenance activity i
R_p	duration of the maintenance of spare part p (in time unit)
A_p	maximum amount of spare parts p
O_{ui}	time interval between last maintenance activity i and beginning of planning horizon for train unit u

3.5 Constants

H	planning horizon
S	shunting cost
k	maximum working load per time unit t (in man-hours)

max_time	maximum working time per time unit t (in hours)
N	number of maintenance activities i (Note: it is the cardinality of the set I)
delay	amount of time needed to move a train from a maintenance line l (in hours)
u_1	maximum number of train units available
u_2	number of train units needed to perform daily service

The parameter k is calculated as the number of men working times the time duration allocated to preventive maintenance per day times the number of working days per time unit t.

The parameter max_time is calculated as the time duration allocated to preventive maintenance per day times the number of working days per time unit t.

3.6 Decision Variables

x_{uitl} binary variable set to 1 if maintenance activity i is performed on train unit u at time unit t, in maintenance line l; and set to 0 otherwise.

y_{ut} binary variable set to 1 if train unit u is under maintenance at t time unit; and set to 0 otherwise.

U_p non-negative integer variable corresponding to the minimum amount of spare part required to perform the technical planning

3.7 Objective function

$$minimize \sum_{u\in U}\sum_{i\in I}\sum_{t\in T}\sum_{l\in L_i} \text{MA\_cost}_i * x_{uitl} + \sum_{u\in U}\sum_{t\in T} S * y_{ut} + H * \sum_{p\in P} \text{SP\_cost}_p * U_p + \frac{1}{(u_1-u_2)*N*H}\sum_{u\in U}\sum_{i\in I}\sum_{t\in T}\sum_{l\in L_i}(H-t) * x_{uitl} \quad (1)$$

Subject to:

$$\sum_{j=t}^{t+T_i}\sum_{l\in L_i} x_{uitl} \geq 1 \quad \forall\, u \in U, i \in I, t \in \{1,\dots,H-T_i+1\} \quad (2)$$

$$\sum_{j=1}^{T_i-O_{ui}}\sum_{l\in L_i} x_{uitl} \geq 1 \quad \forall\, u \in U, i \in I \text{ such that } T_i - O_{ui} \leq H \quad (3)$$

$$y_{ut} \geq x_{uitl} \quad \forall\, u \in U, i \in I, t \in T, l \text{ in } L_i \quad (4)$$

$$\sum_{u\in U}\sum_{i\in I}\sum_{l\in L_i}\sum_{j=t}^{t+R_p} \kappa_{ip} * x_{uitl} \leq U_p \quad \forall\, p \in P, t \in \{1,\dots,H-R_p\} \quad (5)$$

$$U_p \leq A_p \quad \forall\, p \in P \quad (6)$$

$$\sum_{u\in U} \sum_{i\in I} \Delta_i * x_{uitl} \leq k \quad \forall\, t \in T, l \in L_i \tag{7}$$

$$\sum_{u\in U} \sum_{i\in I} duration_i * x_{uitl} + delay * (\sum_{u\in U} \sum_{i\in I} x_{uitl} - 1) \leq \text{max\_time} \quad \forall\, t \in T, l\ in\ L_i \tag{8}$$

$$\sum_{l\ in\ L_i} x_{uitl} \leq 1 \quad \forall\, u \in U,\ \forall\, i \in I,\ \forall\, t \in T \tag{9}$$

x_{uitl} is binary $\forall\, u \in U, i \in I, t \in T, l\ in\ L_i$ (10)

y_{ut} is binary $\forall\, u \in U, t \in T$ (11)

U_p is non-negative integer $\forall\, p \in P$ (12)

The objective function (1) is the total cost of preventive maintenance over a year. It was adapted from an objective function found by Doganay and Bohlin (2010) which fits the objective of minimizing all costs of maintenance of trains in a railway maintenance yard. This function is composed of four different cost components which are i) the maintenance cost - denoted A; ii) the shunting cost - denoted B; iii) the spare parts cost - denoted C and finally iv) a cost to avoid early maintenance - denoted D. The objective function is then A+B+C+D. All the cost components are explained in detail in the following subsections.

Constraint (2) is imposed in order to have each maintenance task i occurring at least once every period T_i for all train units, maintenance tasks and time periods.

Constraint (3) states that every maintenance task i which is due by the end of the planning horizon H is performed at least once.

Constraint (4) imposes that if x_{uitl} is equal to one, i.e. if maintenance activity i is scheduled in a particular time period t for train unit u and line l, then y_{ut} must be equal to one. Therefore, every shunting must be taken into account.

Constraint (5) requires that the number of spare parts needed is greater than the greatest number in service at any single occasion.

Constraint (6) bounds the number of spare parts in order to stay under the limit chosen by the user. This upper bound represents the maintenance yard's storage capacity.

Constraint (7) limits the total working load performed during a time unit t under the maximum amount of work that can be done within one time unit. In this model, the maximum amount of work is not time dependent, which might be changed if the simplification is not relevant.

Constraint (8) makes the maintenance duration on each line stays under the maximum amount of working time per time unit t (time per day times number of working days). A delay, corresponding to the time required to move the trains is added. This delay is multiplied by the total number of movement $\sum_{u\in U} \sum_{i\in I} x_{uitl} - 1$ which is the total number of maintenance activities performed on all the trains minus one; which also is the number of movements on a given maintenance line l.

Constraint (9) imposes that, for each maintenance activity i of train u at a given time t is either not performed (left hand side equal to zero) or performed in a given maintenance line (left hand side equal to 1). The same maintenance activity i on the same train u can only be performed in one maintenance line l.

Constraint (10) makes x_{uitl} a binary variable for all train units, maintenance activities, time units and maintenance lines.

Constraint (11) makes y_{ut} a binary variable for all trains and time units.

Constraint (12) imposes that U_p is a non-negative integer for all spare parts.

3.7.1.1 Maintenance cost A

The maintenance cost A is the cost of doing every maintenance task over the planning horizon. Each maintenance cost MA_cost_i must be given previously as an input; it corresponds to the cost of doing a specific maintenance task i.

The cost component A can be expressed as the sum of all the maintenance costs of the maintenance activities performed on every train, every line and at every time period until the horizon.

$$A = \sum_{u \in U} \sum_{i \in I} \sum_{t \in T} \sum_{l \in L_i} \mathrm{MA\_cost}_i * x_{uitl}$$

3.7.1.2 Shunting cost B

The cost component B is the shunting cost; it corresponds to the cost of pulling a train out of its regular duty in order to perform maintenance on this train. It can be expressed as the sum of the shunting cost per time unit t of all trains stopped every time unit t of the planning horizon.

$$B = \sum_{u \in U} \sum_{t \in T} S * y_{ut}$$

3.7.1.3 Spare part cost C

The cost component C is the cost of keeping spare parts that need to be kept in good conditions even when they are not used. The spare part cost is also defined previously by the user; it is commonly estimated as a percentage of the initial price of the spare part. The cost component C can be set as the product of the duration in time units of the planning horizon and the sum of the spare part cost times the amount of each spare part. It must be highlighted that in this model, minimum amount of spare parts remains the same throughout the year. Therefore, U_p is chosen so that it would fulfil all maintenance activities on all trains and all time periods over the planning horizon.

$$C = H * \sum_{p \in P} \text{SP\_cost}_p * U_p$$

3.7.1.4 Penalty cost D

The last cost component is a term to discourage early maintenance as it is both costly and likely to trigger some early failure of the components. The cost component D can be seen as a penalty if the last preventive maintenance before the end of planning horizon is performed too early. It is the product of $\frac{1}{(u_1-u_2)*N*H}$ which is a weighted penalty, times the distance between the last maintenance perfomed and the end of the planning horizon $(H-t)*x_{uitl}$. The closer to the end of planning horizon the maintenance activity is performed, the smaller the penalty cost is. The weighted penalty is made of the inverse of the product of the total number of maintenance activities, multiplied by the planning horizon times the number of spare trains; i.e. the difference between the number of train units owned by the train operating company and the usefull number of trains to perform daily service.

$$D = \frac{1}{(u_1 - u_2) * N * H} \sum_{u \in U} \sum_{i \in I} \sum_{t \in T} \sum_{l \in L_i} (H - t) * x_{uitl}$$

Chapter 4 Model implementation and validation

In chapter 4, an illustrative example and its implementation is explained in detail in order to provide a better understanding of the mathematical model. In a first subsection, the implementation in FICO Xpress is described. In the second subsection, the parameters of the mathematical model are displayed and in a third subsection the results of the mathematical model are explained.

4.1 Model implementation in FICO Xpress optimization software

Optimization has become a key parameter in nowadays industries as costs and system complexity keep increasing over the years. Maintenance costs represent a major part in the total amount of expenses for most transportation companies. This is why optimization solvers can have a major impact on cost savings. An optimized maintenance is achieved either by maximizing or minimizing at least one function, called ***objective function***. In order to optimize the chosen objective function, an important number of software with optimization solvers can be found online such as Excel; Gurobi; IBM CPlex or FICO Xpress.

"As the premier mathematical modelling and optimization solution in the world, Xpress allows operations researchers, analysts, consultants and others to easily create, deploy and utilize business optimization solutions based on scalable high-performance algorithms, a flexible modelling environment and rapid application [...]." (FICO 2015)

FICO Xpress is a software that enables the user to select different kind of solvers in order to create the mathematical model that will correspond to the real case needing optimization. A non-exhaustive list of solvers that can be chosen is quadratic solvers; nonlinear solvers, mixed-integer linear solvers. In Fertagus case study, the objective function is linear and hence, the constraints are linear as well and the decision variables are integers. This is why a mixed-integer linear solver was selected among the ones available in the software.

Once the solver is selected, the mathematical model itself has to be created. Thus, it is interesting to point out that it has a specific language, called Mosel, needed to write down the required objectives. Indeed, for optimizing one parameter, it is required to write both an ***objective function*** and ***constraints***. The objective function is the expression that Xpress algorithm will either maximize or minimize. The constraints, which restrict the model solution, are the limitations of the problem.

Depending on the parameters and constraints of the problem under analysis, the technical planning would not be the same. Therefore, they have to be adapted for each specific case. The program is divided in four steps which are: first initialization from data files, then the objective function expression, the choice of the constraints of the system and finally the creation of a file with the output values.

The first step of the model (i.e. initialization) sets the parameters of the case study. In order to make every input changes easier for the user; all parameters are controlled through an excel file which is then converted into data files directly read by the program. This way of controlling inputs was chosen because of the size of the problem; it was indeed confusing to declare all parameters directly in the mathematical model.

In order to have an output more user-friendly than simply presenting the values of the decision variable x_{uitl}; the creation of a result file was added in the FICO Xpress program. In order to be able to do this, two parameters are required: MA_type_i and SP_type_p. For every i, MA_type_i is the name of the maintenance activity i, while for every p SP_type_p is the name of the spare part p. Once the optimization calculus is over, the result file reads for each week of the planning horizon the value of the decision variable x_{uitl} for every train u, maintenance activity i and line l. If x_{uitl} is equal to one, then a sentence is written in the result file stating that "MA_type_i has to be performed on line l on train u". Otherwise if x_{uitl} equal to zero, nothing is written in the output file. Furthermore, the second output of the mathematical model is the minimal amount of spare parts needed to perform the technical planning. In order to know easily how many spare parts of each kind are needed, a sentence is written for every spare part type p stating that "Minimal amount of SP_type_p is U_p".

4.2 Parameters of an illustrative example

Model validation consisted in solving a small-scale example and analysing in details its corresponding solution which is presented in this section. In this example, the studied company has 5 trains going to a maintenance yard in which three kinds of maintenances activities can be performed: i_1, i_2 and i_3. Two different spare parts are kept in order to be switched with parts mounted on trains: p_1 and p_2.

The goal of the program is then to find the best technical planning possible, which means the technical planning that will have the smallest cost over a planning horizon of 15 weeks.

Tables 3 to 7 provide values for the parameters used in this mathematical model created to represent this example.

Table 3 : Constant used in the mathematical model of the illustrative example

Constants	Units	Values
H	Weeks	15
S	Monetary units	500
k	Working hours	160
max_time	Hours	40
N	Without units	3
delay	Without units	0.16
u_1	Without units	6
u_2	Without units	5

In Table 3, all the constants of the mathematical model are displayed. First the planning horizon which is 15 weeks, then the shunting cost which is 500 monetary units. The maximum working load per week is 160 working hours and the maximum working time is 40 hours. The maximum working load is calculated by the product of the working time per day times the number of men working per day times the number of useful days in a week. In the illustrative example that would be: k = 8 hours * 4 men * 5 (days) = 160 working hours. The maximum working time per week is simply the maximum working load divided by the number of man working.

Table 4 : Information about maintenance activities

i	MA_type_i	MA_cost_i	T_i (in weeks)	Δ_i (in hours)	$duration_i$ (in working hours)	L_i
1	i_1	80	5	7	3,5	{1,2}
2	i_2	100	30	20	5	{1,2,3}
3	i_3	50	16	11	3,37	{1}

In Table 4, the first line for example is giving information about maintenance activity 1, first the name of the maintenance activity 1 (i_1), then its cost in monetary units (80). The next columns give the period of maintenance activity 1 (5 weeks), the work load (7 working hours), the duration (3,5 hours) and finally the set of maintenance lines where maintenance activity 1 can be performed ({1,2}). Maintenance activity 1 can be done either on line 1 or one line 2 of the maintenance yard.

Table 5 : Information about spare parts

p	SP_type_p	SP_cost_p (per week)	Spare part maintenance duration (in weeks) (R_p)	maximum amount of spare part (A_p)
1	p_1	20	1	20
2	p_2	30	2	20

In table 5, information about the spare parts can be found. For example, the first line gives information about spare part 1, first the name of the spare part (p_1), then cost of the spare part 1 per week (20 monetary units). The next columns give the maintenance duration of the spare part 1 (one week), and the maximum amount of spare part 1 that be stored in the maintenance yard (20 units).

Table 6 : Distance in weeks between the last maintenance activity and beginning of planning horizon for each train unit

Maintenance Activity / Train number	i_1	i_2	i_3
1	4	15	11
2	2	18	1
3	4	12	11
4	3	26	1
5	3	9	8

In table 6, the initial conditions of all trains can be found. In the first line, for example, the initial conditions of train 1 are stated, maintenance activity 1 (called i_1) was performed 4 weeks before the beginning of the planning horizon. Maintenance activity 2 (called i_2) was performed 15 weeks before the beginning of the planning horizon. Maintenance activity 3 (called i_3) was performed 11 weeks before the beginning of the planning horizon.

Table 7 : Number of spare part used for each maintenance activity

Maintenance Activity / Spare part	i_1	i_2	i_3
p_1	1	0	1
p_2	0	1	1

In table 7 displays which kind of spare part is used for which maintenance activity. In the first line the following indications can be seen, one spare part 1 (called p_1) is needed to perform maintenance activity 1 (called i_1); no spare part 1 is needed to perform maintenance activity 2 (called i_2). Finally, one spare part 1 is needed to perform maintenance activity 3 (called i_3).

4.3 Results of the optimization model

Once the algorithm converges to a solution, the program displays the minimum cost found for the technical planning over 15 weeks, as well as the minimum number of spare parts required to fulfil the technical planning (Figure 2). A data file with the technical planning inside is created, and enables to build the planning shown in Table 8.

Table 8 : Technical planning of the illustrative example

Week number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Train 1															
i_1	X					X					X				
i_2											X				
i_3	X														
Train 2															
i_1			X					X					X		
i_2								X							
i_3													X		
Train 3															
i_1	X					X					X				
i_2															
i_3	X														
Train 4															
i_1		X					X					X			
i_2		X													
i_3												X			
Train 5															
i_1		X					X					X			
i_2															
i_3							X								

From the technical planning, several facts can be highlighted. First, it can be seen that the period of the maintenance activities is respected if nothing interferes. For train 3 for example, maintenance activity i_1 is performed every 5 weeks as required by the user inputs (on Table 4 : Information about maintenance activitiesTable 4). The period can be shorter when another maintenance task is scheduled few weeks before the optimal date in order to share the shunting costs. It can be seen for train 2, when maintenance i_2 is performed on week 8, which is four weeks ahead of the deadline.

Interestingly, no maintenance 2 was performed for train 3 as the period of this maintenance is set to be 30 weeks, and it was performed 12 weeks before week 1. This implies that, during the next planning horizon, train 3 will probably go to under maintenance 2 at most on week 3 (30-12-15 =2).

The first maintenance activity i_1 of train unit 2 is performed on week 3 as it was done two weeks before the beginning of the planning horizon; i_1 is then performed on week 8, after five weeks – which is the period of i_1. Train 2 is under maintenance i_2 on week 8; instead of week 12 if the period of 30 weeks was strictly followed (30–18 =12). Indeed, i_1 is to be performed on week 8 as well and shunting costs can be shared if the two maintenance activities are performed together. Of course, doing i_2 on week 13 with i_1 could be tempting as it is closer to week 12, but i_2 would then be performed one week late which is impossible.

```
The minimum cost is : 11050.8
Maximum amout of p1 : 3
Maximum amout of p2 : 2
```

Figure 2 : Total cost and amount of spare parts of the illustrative example

In order to be able to fulfil the optimized technical planning; three spare parts p_1 and two spare parts p_2 are needed as pointed out in Figure 2. The number of spare part is not varying depending on time because it is assumed that no spare parts are bought during the planning horizon.

The maintenance line number is also chosen within the possible set L_i specified by a user input. In the example, i_1 can be performed on lines 1 and 2 which means that the model indicates that the maintenance activity is to be done either on line 1 or line 2. In the Figure 3 below is giving the assignments for the first week of the technical planning. In this file, information can be found about which train is going under maintenance on a given line l of L_i.

```
Week #1
i1 has to be performed on line 1 for train 1
i1 has to be performed on line 2 for train 3
i3 has to be performed on line 1 for train 1
i3 has to be performed on line 1 for train 3
```

Figure 3 : First week of the example's technical planning

As the size of the problem is relatively small, it can be seen that the time duration for the model to get to a nil gap is a tenth of a second (Figure 4). It must be said that computational time will increase if either the number of train units increases or the number of possible maintenance activities, spare parts or lines is larger.

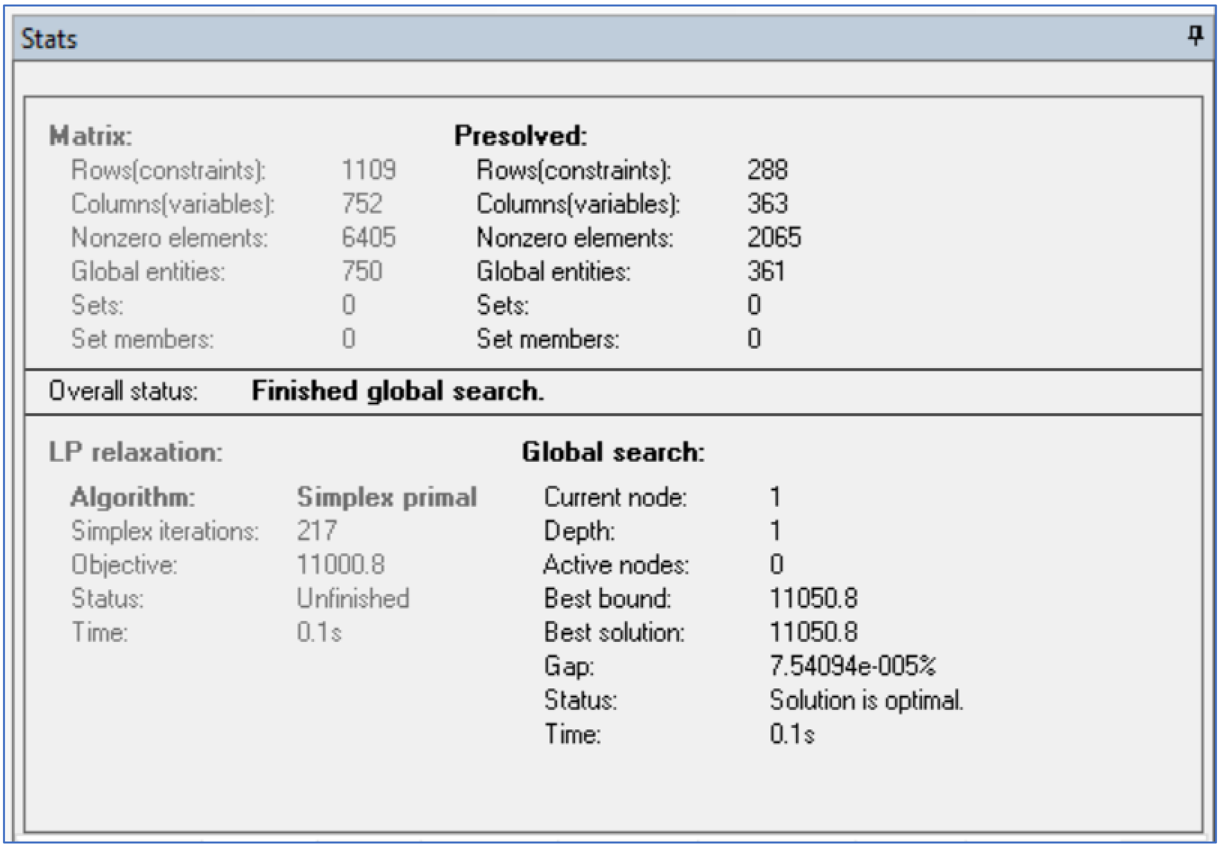

Figure 4 : Calculation duration

Figure 5 is an output from FICO Xpress software that delivers useful information such as the size of the matrix before (Table 9) and after pre-solving stage, as well as information on the final solution (Table 10).

Table 9 : Explanation of the calculus of the column of the matrix

Set	Size
u={1,…,5}	5
i={1,…,3}	3
t={1,…,15}	15
l={1,…,3}	3
p={1,2}	2

decision variables	size
x_{uitl}	5*3*15*3=675
y_{ut}	5*15=75
U_p	2=2

Size of the columns
675+75+2=752

The matrix column's size is calculated thanks to the size of each set of the mathematical model. All decision variables have a size which is the product of the size of the sets of their corresponding indices. For example, x_{uitl} is made of the variables u, I, t and l, hence its size is the product of the size of variables. In total, the matrix is made of 752 columns before pre-solving stage. This stage enables to supress some of the redundancy within the initial matrix which reduces its size. The pre-solving process is specific to FICO Xpress and is not accessible to the user.

Table 10 : Solution information

Solution information	Value
Best bound	11050,8 monetary units
Best solution	11050,8 monetary units
Gap	0 %
Computational time	0,1 seconds

In Table 10, all the useful information about the solution that was found are given. The best bound of the objective function is in this case the best lower bound, as this objective is to find a minimum. In the illustrative example, it is interesting to underline that the solution is optimal because the optimality gap is zero. The optimality gap is indeed the ratio, converted in percentage, of the difference between the solution of the total cost and the best bound of the cost.

$$optimality\ gap = \frac{total\ cost\ solution - lower\ bound}{total\ cost\ solution} * 100$$

In this illustrative example, the computational time to achieve a nil gap was a tenth of a second because the size of the problem was small. However, in larger size problems the computational time increases and the optimality gap is a good indicator of the optimality of the solution when the computation is stopped. The smaller the optimality gap, the closer the solution is to the optimal value.

Chapter 5 The Case study of Fertagus train operating company

In chapter 5, Fertagus train operating company is presented briefly and problem specifications are introduced. The parameters of mathematical model are displayed in a table with all cost values given in monetary units for the sake of confidentiality.

5.1 Fertagus train operating company

As depicted in Figure 5, Fertagus trains run on a line of 54 kilometres that crosses the "*25 de Abril"* bridge and stop at 14 stations. Total travel duration between Roma-Areeiro and Setúbal is 57 minutes and the bridge crossing is only 7 minutes long.

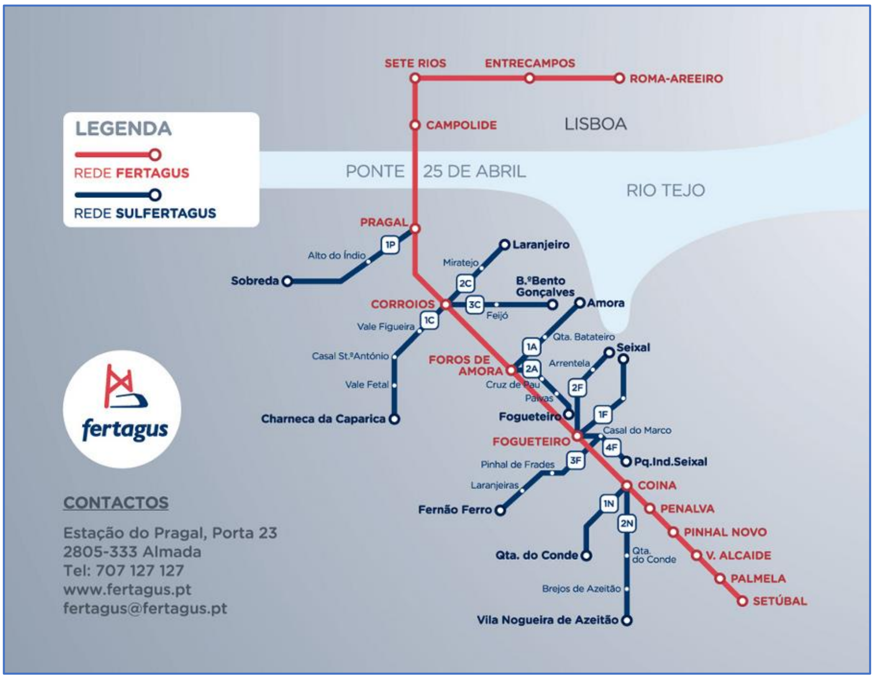

Figure 5 : Fertagus' railway map (source: Fertagus website)

Fertagus is a train operating company whose name derives both from "*caminhos-de-ferro*" and the Tagus river's name. It became the first private rail operator in Portugal when it won the call for bids for the line between Lisbon's city centre and the Setubal district area. As a result, it now has a contract that ensures availability, cost and duration of travels. As it was the first private railway operator in Lisbon metropolitan area, it is interesting to realize that availability is part of the contract with Lisbon's city centre. Indeed, in order to optimize maintenance, it must be kept in mind that no train can be pulled out of service to go to maintenance if there is no backup train available. In order to guarantee that this would not happen, Fertagus chose to have eighteen trains when only seventeen are necessary to perform the current operation schedules. The question of whether or not this could be done differently is out of the scope of the present work and is left for further research. However, it might not be necessary to have an additional train if instead of doing mileage-based maintenance, Fertagus was using a condition-based maintenance approach.

It must be said that as Fertagus does not own the railway line, infrastructure charges must be paid to the infrastructure manager, IP, and the railway infrastructure maintenance is not up to Fertagus. This also means that Fertagus trains are not the only trains running over the line between Roma-Areeiro and Setubal, which might result in technical issues as not all trains have the same requirements. Indeed, during one of the visits to the maintenance yard, Eng. João Duarte told us that since the line started to be used by other trains going faster, unusual wear was noticed on all the trains' wheelsets. Nevertheless, this might also be due to the wheelsets that have replaced the old ones. A more detailed analysis is needed and is also left to further research.

During this work, three meetings with Fertagus staff were scheduled and some additional information was given by email and phone calls. This work benefited from two main contacts, Engineer João Grossinho and Engineer João Duarte in the maintenance yard of Fertagus.

Fertagus maintenance yard comprises 10 lines and its division configuration can be seen in Figure 6. Although they are numbered from 1 to 12, lines number 3 and 4 were never built but were designed in the original maintenance yard's plans. Out of these lines, only 3 are used to perform maintenance tasks, respectively 10, 11 and 12. The other lines are used for testing, parking or cleaning operations. A summary of all lines' current use can be found in Table 11 below.

Table 11 : Lines of Fertagus maintenance yard

Line number	Use in the maintenance yard
1	Several tests
2	Several tests
5	Parking
6	Parking
7	Parking
8	Parking
9	Cleaning operations & conservation cleaning
10	Maintenance with catenary
11	Maintenance with catenary
12	Maintenance without catenary (includes pantograph replacement)

Figure 6 : Plane view of Fertagus maintenance yard

In Figure 6, a plane view of Fertagus maintenance yard is shown. It can be seen that all lines used to perform maintenance activities (lines 10, 11 and 12) are covered by a roof in order to avoid rain falling on trains while they are being maintained. On the contrary, line number 9 which is dedicated to cleaning is not covered because this maintenance activity is to be performed outside; this line is located on the left of the maintenance yard. Fertagus maintenance yard also performs wheelset turning within the maintenance yard in an underfloor room that is shown on the figure above.

5.2 Specific input parameters for MILP formulation

In order to model the present case study, some information about the maintenance activities was gathered in order to have the correct inputs for the parameters of the mathematical model. During meetings, Fertagus maintenance activities were explained by Eng. João Grossinho and Eng. João Duarte and summarized on the tables below. While the first one corresponds to the maintenance activities scheduled by Fertagus maintenance crew; the second table sums up the maintenance activities scheduled by a consultancy company which provides support for major renewals. Because of the way the program was built, only the maintenance activities scheduled by Fertagus maintenance crew could be taken into account. The integration of R1, R2 and R3 major renewals in the mathematical model is left for further research.

Table 12 : Maintenance tasks which are not performed by Fertagus crew

Maintenance activity	**Tasks performed during MA**	**Period**	**Time needed to perform MA**	**Men force required**	**Cost**
R3	Pantograph are replaced	Every 600,000 km	Unknown	Unknown	Unknown
R2	replacement of wheelsets, bogies are repaired and sent to EMEF or RENFE	Every 1,200,000 km	Unknown	Unknown	Unknown
R1	replacement of wheelsets, bogies are repaired and sent to EMEF or RENFE	Every 1,800,000 km	Unknown	Unknown	Unknown

Table 13 : Maintenance tasks performed by Fertagus crew

Maintenance activity	Tasks performed during MA	Period	Time needed to perform MA	Men force required
ETS *"Ensaios e Trabalhos Sistemáticos"*	Mostly inspection activities	Every 5 weeks	1h30-2h00	4
VEq *"Visita de Equipamento"*	Inspection of motor block, pressure check, etc.	Every 37,500 km	6h00	4
VP *"Visita de Portas"*	Doors check-up	Every 150,000 km	6h00	4
VL *"Visita de Lubrificação"*	Lubrication check-up	Every 120,000 km	4h00-6h00	4
VEl *"Visita Eléctrica"*	Electric system check-up	Once a year (before winter)	40h00 (not continuous)	4
VS *"Visitas Sazonais"*	Biannual check-up	Twice a year (beginning of Spring and end of Summer)	12h00	4
TRF *"Torneamento dos rodados"*	Wheelsets turning	Every 120,000 km	16h00 during weekend	2
V1	Some parts of the pantograph are maintained	Every 300,000 km	Unknown	4

The mathematical model's parameters for the Fertagus case study were extracted from the two tables above and completed through additional meetings; emails or phone calls. All parameters used for the Fertagus case study can be found in the following tables (Table 14, Table 15, Table 16, Table 17, Table 18 and Table 19).

Table 14 : Sets of the mathematical model

Sets	Values
U	{1,...,18}
I	{1,...,16}
T	{1,…,53}
P	{1,…,4}

In Table 14 the sets of Fertagus case study are displayed; they are eighteen trains so U is a set of integers going from 1 to 18. Furthermore, sixteen maintenance activities can be performed in Fertagus maintenance yard which implies that i is a set of integers from 1 to 16. The planning horizon of the technical planning is a year so, since the time unit is a week, T is a set of integers from 1 to 53. It is true that a year is made of less than 53 weeks, but as it is also more than 52 weeks; the horizon was chosen to stop at week 53 in order to include the few days remaining after week 52. Finally, 4 different spare parts are stored in Fertagus maintenance yard so P is a set of integers from 1 to 4.

Table 15 : Parameters of the mathematical model depending on the maintenance activity i

i	MA_type$_i$	MA_cost$_i$	T_i (in weeks)	Δ_i (in working hour)	duration$_i$	L_i
1	ETS	614,42	5	10	2,5	{10 11}
2	VEQ	1720,37	16	28	7	{10 11}
3	LUB	1018,98	53	14	3,5	{10 11}
4	POR	829,17	63	14	3,5	{10 11}
5	TRF	2522,22	63	42	21	{10 11}
6	LL	815,28	63	14	4,6	{10 11}
7	ELET1	3516,44	53	12,4	3,1	{10 11}
8	ELET2	3516,44	53	12,4	3,1	{10 11}
9	ELET3	3516,44	53	12,4	3,1	{10 11}
10	ELET4	3516,44	53	12,4	3,1	{10 11}
11	ELET5	3516,44	53	12,4	3,1	{10 11}
12	AC	793,29	53	14	7	{10 11}
13	BAT1	396,64	53	3,5	0,88	{12}
14	BAT2	396,64	53	3,5	0,88	{12}
15	MR	56,25	26	1	1	{10 11}
16	V1	2457,68	136	40	10	{12}

In Table 15 all parameters depending on the maintenance activities i are summarized. The first line includes the name of maintenance activity 1 which is ETS, its cost in monetary units which is 614,42. Then the period of the ETS maintenance is displayed in weeks and is equal to five weeks. This means that maintenance activity 1 (called ETS) is due every five weeks. Then, both the working load and the duration of the maintenance activity 1 are given. ETS maintenance is a 10 working hours maintenance activity and lasts 2,5 hours long. Because the working load and the duration are linked by the relation "working load = duration * working men", it can be deduced that currently 4 men are needed to do maintenance activity 1. Finally, the set of maintenance lines where maintenance activity 1 can be performed is displayed. It can be read that maintenance ETS can be performed either on line 10 or on line 11 of Fertagus maintenance yard. Indeed, it was explained in Table 11 that line 10 and 11 are equipped with the same tools and can therefore be used for the same maintenance activities.

Table 16 : Parameters of the mathematical model depending on the spare parts p

p	SP_type$_p$	SP_cost$_p$	R_p	A_p
1	wheelset	104,17	1	20
2	trailer bogie	1041,67	0	20
3	motor bogie	1041,67	1	20
4	pantograph	416,67	2	20

Parameters that depends on the spare parts p are displayed in Table 16. In the first line, all information about the spare part 1 is given; the first piece of information is the spare part's name which is a wheelset. Then, the cost of the spare part 1 per week is given in monetary units and is 104,17. The next parameter is the number of weeks needed to maintain the spare wheelset and its value is one which means that when a spare wheelset is sent to maintenance it will be unavailable for one week. Finally, the maximum number of spare parts 1 is given according to the maintenance yard storage area. Because Fertagus maintenance yard is relatively large, it is assumed that storage would not be an issue; this is why the maximum number of wheelsets is set to 20. Here the largest number of spare parts does not constraint the solution, it only reduces the number of values that the solver will test. Thus, it reduces the computational time.

Table 17 : Time interval between last maintenance activity i and beginning of planning horizon for train u

Train number\MA	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	4	3	19	50	46	25	40	40	40	40	40	13	16	16	19	45
2	2	5	26	7	42	25	40	40	40	40	40	13	16	16	9	40
3	4	0	37	19	20	24	36	36	36	36	36	12	16	16	16	54
4	3	10	40	30	15	20	35	35	35	35	35	12	16	16	18	56
5	3	15	16	16	15	24	15	15	15	15	15	12	15	15	24	56
6	0	14	2	42	36	23	14	14	14	14	14	11	15	15	3	54
7	3	15	18	23	49	23	13	13	13	13	13	11	15	15	17	47
8	0	1	30	30	51	22	12	12	12	12	12	10	14	14	4	55
9	1	14	38	14	47	22	12	12	12	12	12	10	14	14	11	46
10	1	0	5	37	39	21	11	11	11	11	11	9	14	14	0	35
11	3	9	43	16	44	21	10	10	10	10	10	9	13	13	5	41
12	4	11	52	45	43	20	7	7	7	7	7	8	13	13	6	35
13	2	10	17	3	45	20	6	6	6	6	6	8	13	13	18	41
14	2	14	13	48	46	19	5	5	5	5	5	7	12	12	16	35
15	4	6	33	22	40	19	4	4	4	4	4	7	12	12	8	49
16	0	1	46	45	36	18	3	3	3	3	3	6	12	12	23	37
17	3	15	39	55	48	18	2	2	2	2	2	6	11	11	20	43
18	4	12	28	35	35	17	1	1	1	1	1	5	11	11	14	44

Initial conditions about Fertagus trains can be found in Table 17. The different maintenance activities are put in the columns while the different lines correspond to different trains. The first line set all the initial conditions for train 1. The first value is the distance between the last maintenance activity 1 and the beginning of the planning horizon and is 4; which means that the last maintenance activity 1 (called ETS) was performed on train 1 four weeks ago. It must be highlighted that all values of O_{ui} can be deduced from this table. For example, O_{23}, which corresponds to the last time maintenance activity 3 was performed on train 2, is set to 23 weeks in the above table. Therefore, maintenance activity 3 was done 23 weeks before the beginning of the planning horizon.

Table 18 : κ_{ip} parameter of the mathematical model

SP\MA	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Table 18 summarizes the values of parameter κ_{ip} which is the amount of spare part p required to perform maintenance activity i. Sixteen different maintenance activities can be done in Fertagus maintenance yard; but they do not all require the same spare parts. Most of Fertagus maintenance activities, such as maintenance activity 1, do not need any spare part. This can be seen on the first column of the above table, where all values are set to zero. It must be said that spare parts are mostly used during corrective maintenance in Fertagus case study which is why the cost of preventive maintenance is lower. However, some maintenance activities of Fertagus railway operating company still need spare part to be done. It is the case of maintenance activity 5 that requires one spare part one (which is a wheelset) to be performed.

Table 19 : Constants of the mathematical model

Constants	Units	Values
H	weeks	53
S	monetary units	5000
k	working hours	160
max_time	hours	40
N	without units	17
delay	hours	0.16
u_1	without units	18
u_2	without units	17

All constants of the mathematical model can be found in Table 19. First the planning horizon H whose value is 53 weeks. It is true that a year is made of less than 53 weeks, but as it is also more than 52 weeks; the horizon was chosen to stop at week 53 in order to include the few days remaining after week 52 The shunting cost is then set to 5000 monetary units, this value was initially given as an approximation and will thus be the subject to a sensitivity analysis in the next chapter. The maximal working load k which is 160 working hours is calculated as the product of the number of men working in Fertagus maintenance yard by the number of working hours per day times the number of useful days of the week. In the case study it is then, 4 men * 8 hours * 5 (days) = 160 working hours. The maximal working time per week is 40 hours and is calculated as the product of the number of working hours per day times the number of useful days in a week. In Fertagus case study it is 8 hours * 5 (days) = 40 hours. It is interesting to notice that the maximal working load and the maximal working time are related to one another with the equation working load = working time * number of men.

Chapter 6 Results and Discussion

In chapter 6 several studies can be found. Firstly, an analysis of the optimality gap over calculation time. In the next subsections, a sensitivity analysis of both the shunting cost component and the maximal working time per week.

6.1 Results of Fertagus case study

This section displays the result of the optimization of Fertagus case study after one hour computation. It must be said that the mathematical model had been stopped before it could reach an optimal solution. In the next subsections, the influence of several parameters will be studied, but the reference case is the following one. The output data about the matrix from the software can be found in Figure 7.

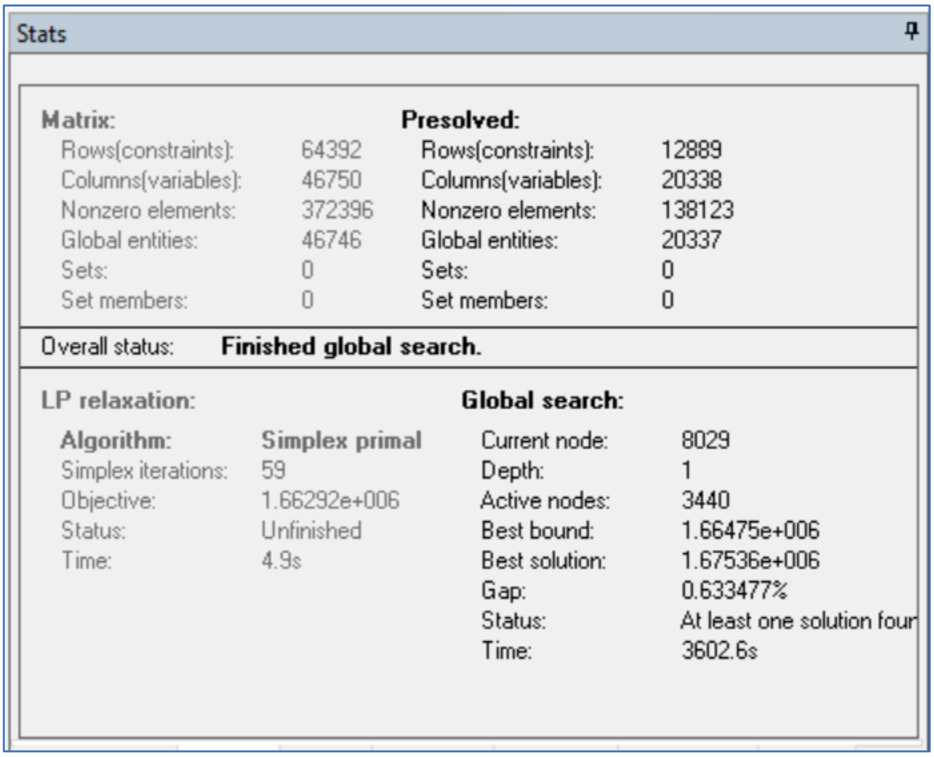

Figure 7: Computation data of Fertagus case study

In Figure 7 above, the same output of computational data as in the illustrative example can be find. However, the case study has a much larger size than the illustrative example. Indeed, the matrix has 46750 columns, which is sixty times more than the illustrative example. With a larger matrix, the computational time to get an optimal value is expected to increase; indeed, after one hour the solution for the total cost of preventive maintenance is still not optimal but has an optimality gap of 0,63%. An analysis of the optimality gap as a function of calculation time is done in the next subsection.

The best solution is a preventive maintenance cost of 1664750 monetary units; this cost is divided into four cost components A, B, C and D whose values are displayed in Table 20.

Cost component	Value (monetary units)	Percentage of the total cost
A (maintenance activity cost)	670495	39,89%
B (shunting cost)	1005000	59,78%
C (spare part cost)	5521,01	0,33%
D (penalty cost)	15,5	0,0009%

Table 20: Values of the cost components

In Table 20, the cost components percentage can be seen. The cost component B is the one with the largest impact on the total preventive cost which is why maintenance activities are grouped whenever it is possible. The cost component A, which is the maintenance activity cost, has the second largest impact. However, it is not a cost that is easily changed as it is dependent on the maintenance activities themselves. The cost components C and D have smaller influence on the total preventive maintenance cost.

6.2 Analysis of the optimality gap as a function of calculation time

As it was said in chapter 3, when the size of the problem increases, so does the computational time to get to the optimal solution. Indeed, if optimality of the solution is not taken in consideration, then a feasible solution can always be found within few minutes. However, if optimality of the solution is important, the gap corresponding to the feasible solution is an indicator of the optimality of the solution. The closer the gap is to zero, the better the solution is. When the size of the problem is relatively small (such as the illustrative example that has 752 variables), the optimal solution can be found in a tenth of a second as it was the case in the illustrative example. Nevertheless, in the case of Fertagus the computational time is much larger, due to a significant increase of the size of the sets. It is then interesting to study the evolution of the gap with respect to computational time in order to know when the solution can be considered as satisfying. Indeed, most of the time the calculus is stopped before the exact solution is reached (i.e. when the gap is 0) in order to save computational time.

In this analysis, the computational time was increased from 1 minute to 24 hours, and the graph of gap versus computational time can be found in Figure 8. The goal of this study is to be able to select the smallest computational time, whose corresponding optimality gap is acceptable. It can be seen on the graph that after a calculation time of one hour, an optimality gap of around 0,6% is achieved. When computational time is increased to one day, the gap becomes slightly less than 0,5% which is better but may not be worth the additional time spent to minimize the cost. Consequently, the computational time was chosen to be set to one hour for all further analysis in this chapter. It is of course required to stay on the same machine for all computation, as the computer itself influences the results.

The detailed values of both the computational time and the associated gap can be found in Figure 8 and Table 21.

Table 21 Values of computational time and corresponding optimality gap

Computational time (s)	Optimality Gap (%)
61 (1 min)	2,67
304 (5 min)	2,67
601,4 (10 min)	2,30
913,5 (15 min)	1,55
1798,5 (30 min)	0,97
2701,2 (45 min)	0,97
3602,6 (1 h)	0,63
5411,9 (1h30)	0,63
18014,6 (5 h)	0,63
36006,9 (10 h)	0,63
86405 (24 h)	0,60

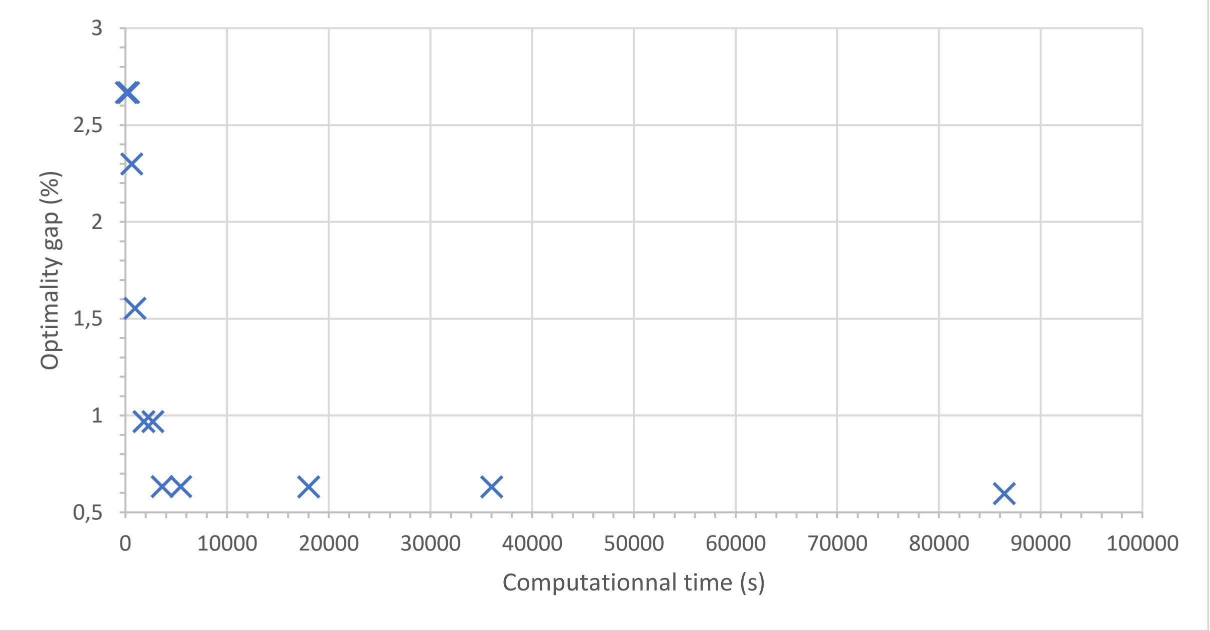

Figure 8 Graph of optimality gap with respect to computational time

In can be seen in Figure 8 that the variation of the optimality gap is a lot faster at the beginning. Indeed, during the first hour, the gap decreases from 2,67% to 0,63%, while in the remaining twenty-three hours it only decreases from 0,63% to 0,6%. This graph proves that the longer the computational time, the smaller the optimality gap but it must be highlighted that the evolution is not linear.

6.3 Analysis of maintenance costs as a function of shunting cost component

In Fertagus case, the shunting cost component value is set to 5000 monetary units. However, this cost was provided as an approximated value so it was worth doing a sensitivity analysis on this parameter of the mathematical model. In order to study the influence of the parameter on the total cost of the maintenance, the shunting cost component is varied from 4500 monetary units to 5500 monetary units. This corresponds to an increase and a decrease by 10% of this cost component.

The graph of the total maintenance cost with respect to the shunting cost component can be found in Figure 9. Another graph can be found in Figure 10 representing the variations in percentage of the total maintenance cost induced by the variations in percentage of shunting cost component.

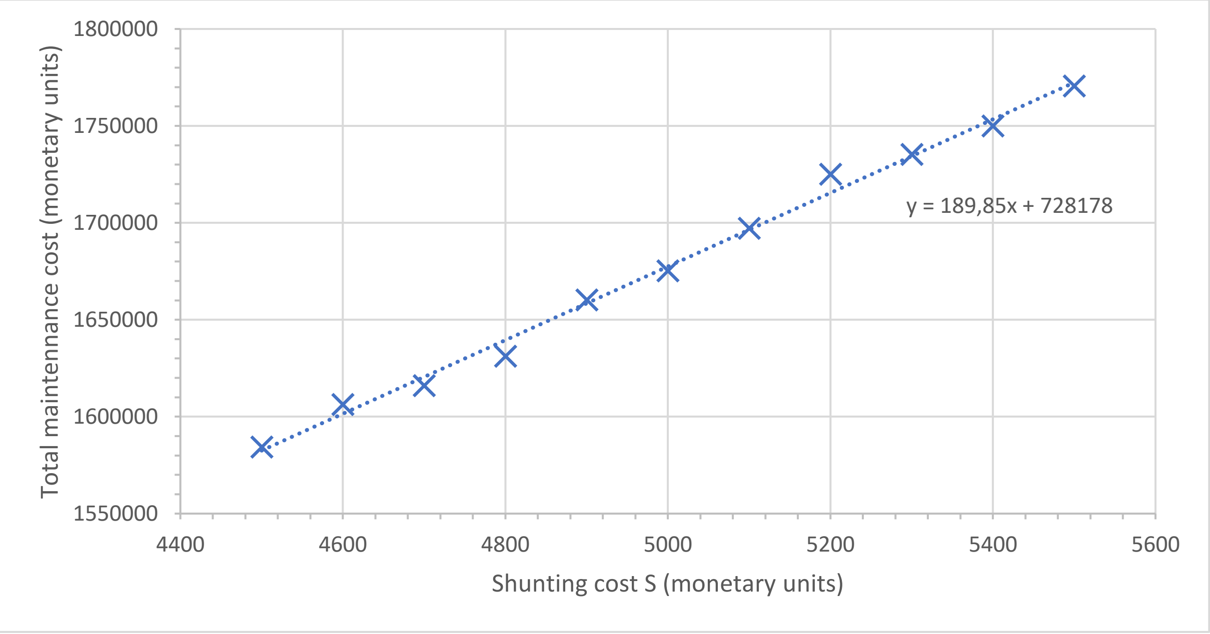

Figure 9 : Total maintenance cost versus shunting cost component

It can be seen in Figure 9 that the evolution of the total preventive maintenance cost is similar to a linear evolution between 4500 and 5500 monetary units. The linear curve displayed on the above figure has an equation that is y =18985x + 728178; this linear function values has a difference with the real value of 0,2% in average.

Table 22 : Relation between the two variations

Shunting variation (%)	Total maintenance variation (%)	Relation between the two variations
-10	-5,43	0,54
-8	-4,12	0,51
-6	-3,54	0,59
-4	-2,63	0,66
-2	-0,90	0,45
0	0,00	Not relevant
2	1,30	0,65
4	2,97	0,74
6	3,58	0,60
8	4,46	0,56
10	5,68	0,57

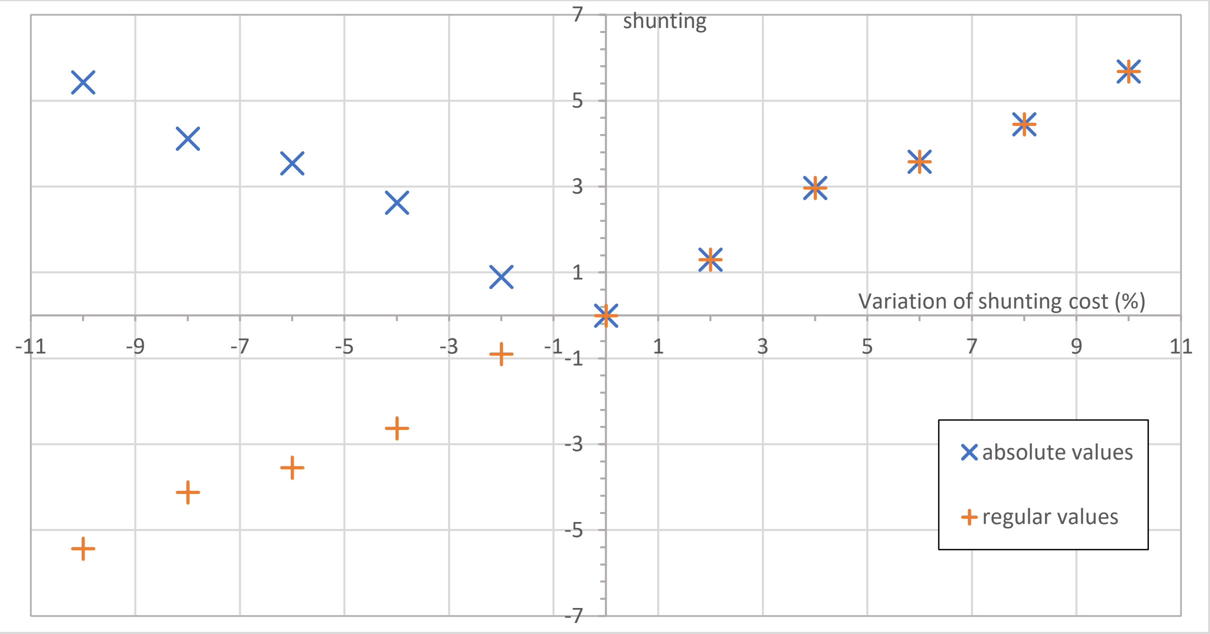

Figure 10 : Variation of total cost with respect to shunting cost component variations

Two different curves can be found in Figure 10, one is called "regular values and the other is "absolute values". The first one, with "regular values", corresponds to the variation in percentage of the total maintenance cost with respect to the variation in percentage of shunting cost (between the reference value 5000 and the value used in the mathematical model). The other curve the "absolute values" corresponds to the absolute value of the variation in percentage of the total maintenance cost, with respect to the variation in percentage of shunting cost.

A variation of the shunting cost of 2% induces a variation of total maintenance variation of 1,3% which corresponds to a relation of 0,65 between the total maintenance variation and the shunting cost variation. In Table 22 a summary of the relations between shunting cost component and total maintenance variations. The average value of these relations is 0,58 which means that the variation of shunting cost component induces a smaller variation of the total maintenance cost.

It is interesting to notice that the “absolute values” curve in Figure 10 : Variation of total cost with respect to shunting cost component variationsFigure 10 is nearly symmetric with respect to the y-axis. It means that an increase of 3% of the shunting cost component will induce a raise on the total cost; and this raise has the similar absolute value than the diminution of the total costs induced by a decrease of 3% in the shunting cost component. It must be said that the values induced by a negative shunting variation are always slightly higher than the ones induced by a positive shunting variation. For instance, a variation of the shunting cost of -2% induces a total cost variation of 0,90% while a variation of shunting cost of +2% induces a total cost variation of 1,30%

The relation between the shunting variation in percentage and the total maintenance variation in percentage can be found in Table 22. The value for a variation of zero percent is not considered to be relevant as a division by zero would be involved otherwise. It is interesting to highlight that all the ratio between the total maintenance variation and the shunting variation are always smaller than one which means that the variation of total cost is damped with respect to the shunting cost variation.

6.4 Analysis of total maintenance costs as a function of working time per week

Fertagus preventive maintenance is done in a shift of 8 hours per day, and five days a week; which corresponds to a current maximum working time in the maintenance yard of 160 hours/week. In order to quantify the impact of the time allocated to preventive maintenance, an analysis was performed in order to assess the evolution of total maintenance costs with respect to the evolution of the maximum working time. It must be highlighted that even if only the maximum working time value changes, it affects two parameters of the mathematical model which are k and max_time (the maximum working load per week and the maximum working time per week).

The graphs in Figure 11 and Figure 12 show the variation of the total maintenance costs and the optimality gap as a function of the variation of the time allocated to preventive maintenance per week. The time allocated varies from 144 working hours to 176 working hours; 160 working hours being the reference (all values can be found in Table 22). When the time allocated is 144 working hours, there is no value for the gap nor for the total maintenance costs because no feasible solution could be found. On the contrary when the time allocated is 176 working hours, the optimal solution is find in 2445,8 seconds (40min and 45 seconds) which means the calculus stops before the end of the reference computational time. For an allocated time of 168 working hours the solution is optimal after 2684 seconds (44 min and 46 seconds).

Table 23 : Values of gap and total maintenance costs for different time duration

allocated working hours per week	allocated hours per week	gap after one hour (%)	total maintenance cost (monetary units)
36	144	-	No feasible solution
38	152	1,84	1701030
40	160	0,63	1675360
42	168	0	1659800
44	176	0	1659750

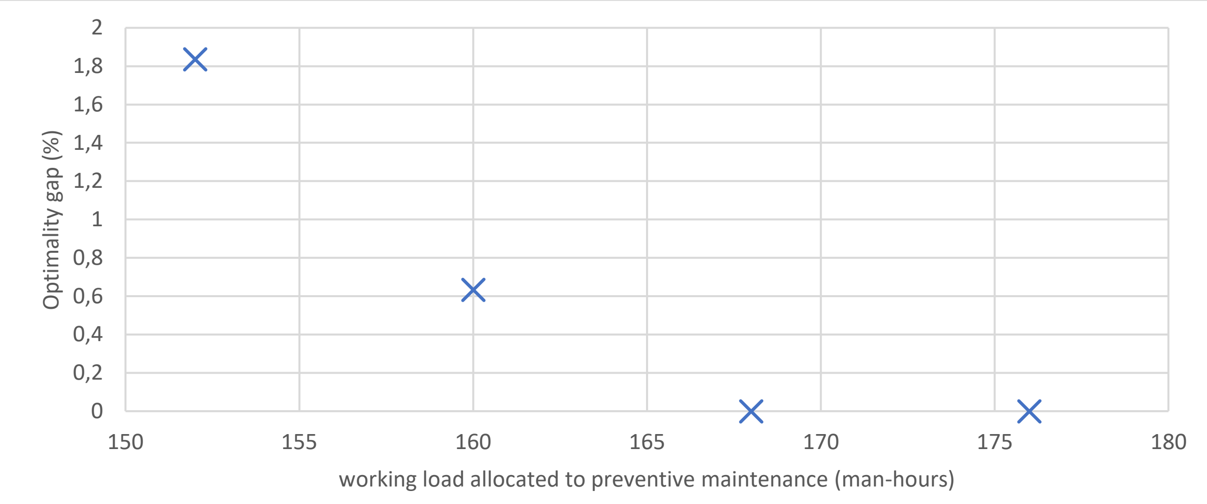

Figure 11 : Graph of optimality gap versus time allocated per week

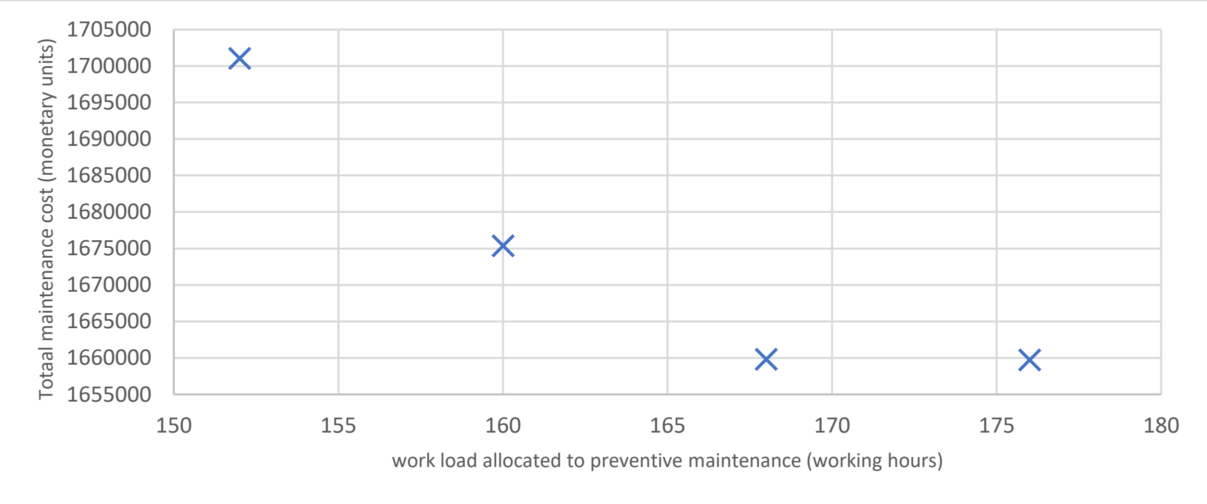

Figure 12 : Graph of total maintenance costs versus time allocated per week

The horizontal axis of Figure 11 and Figure 12 are the same, which is logical considering that the working load allocated and the working time allocated are related through the equation:

load = working time * number of men. During the sensitivity analysis, the number of men does not change which explains why only one parameter was used to plot the graph of the sensitivity analysis.

On Figure 11 it can be noticed that the optimality gap value is increased three times when the working load is lower than 160 working hours. If the working load is lower than 152 working hours, something curious appears, there is no feasible solution that can be found. There are two possible explanations for this; either 160 working hours working load for preventive maintenance is considered optimal in Fertagus case study; or the initial conditions have a lot of influence on the technical planning. Indeed, a maintenance which is done by four working men for several planning horizons could lead to initial conditions that require a preventive maintenance done by four men. Therefore, after having performed preventive maintenance a certain way for a long time it could be difficult to change the way to do things.

On Figure 12, the total cost of preventive maintenance is decreasing when the working load increases. Indeed, maintenance activities that have to be performed on a given week are the same no matter the allocated working load. However, it is easier to schedule maintenance activities if the working load is not restrictive. This is why the total cost of preventive maintenance is lower for a working load of 176 working hours than for a working load of 168 working hours; even if the solution is optimal in both cases. On the contrary, if the working load is too low it may be impossible to schedule maintenance activities, it is the case when the working load is 144 working hours.

Chapter 7 Conclusions and Future Research

In the final chapter, the conclusion of the performed research can be found as well as some limitations and possible further steps of the future research.

7.1 Conclusions

Optimizing the total costs of preventive maintenance is of course the objective of every company, since it would have a non-neglectable effect on the budget. The goal of this dissertation was to create a mathematical model that would provide an optimal technical planning, reducing the total preventive maintenance costs to a minimum. The mathematical model created was adapted to the specific case of Fertagus railway operator but can be very easily be modified to fit to any company's specifications. This adaptation involved collecting data related to maintenance activity operations in the maintenance yard and some sensitive costs which are displayed in monetary units for the sake of confidentiality.

One of the objectives of this dissertation was to prove that this program would give the optimal feasible technical planning if at least one can be found. It is important to underline that whenever a parameter was too restrictive, the mathematical model would say that no feasible solution could be found. Several simulations were made in order to quantify the sensitivity of some parameters and the mathematical model was considered satisfying.

Moreover, it is interesting to realize that the mathematical model is able to give a feasible solution with an optimality gap of less than one percent in less than one hour. Indeed, most company would care about the optimality of the solution. This is why, even if a technical planning is performed once a year, it is still compelling that the computational time for a Fertagus size stays relatively low.

7.2 Limitations

This mathematical model enables to find an optimal technical planning but it is of course user input dependent and this is a major limitation. Indeed, if the user inputs do not represent correctly the real-life situation, the technical planning could hardly be optimal, even if an optimal solution is found. This is the case in Fertagus maintenance yard, which means that the solution found is actually not the best possible. Indeed, some maintenance activities are performed by Fertagus maintenance crew, but some others (such as R1, R2 and R3) are scheduled by a consultant company. These maintenance activities were not taken into consideration and consequently the total maintenance cost found by the program might not be the actual optimal one.

7.3 Future Research

As mentioned in the previous subsection, all maintenance activities scheduled by a consultancy company are not taken into account. In order to provide a total maintenance costs which would be more accurate, a new cost component will have to be added. This cost component should reflect the cost of having these maintenance activities done outside of Fertagus maintenance yard. It should be made of both a cost of unavailability and a labor cost and will have to be added to the mathematical model.

Moreover, even if this dissertation provides a first approach of an optimized technical planning, it must be kept in mind that the cost of preventive maintenance represents barely a half of the costs of corrective maintenance. This means that if a company really wants to save costs on maintenance, corrective maintenance should really be taken into consideration. In Fertagus case, corrective maintenance is done every day during eight hours a day, right after the eight hours dedicated to preventive maintenance. One way of taking into account corrective maintenance would be to add a corrective maintenance cost component in the mathematical model. However, ideally a new program should be created since corrective maintenance can only be predicted through predictive model while preventive maintenance has fixed period.

Finally, it must be said that obtaining an optimal technical planning is only the first step. After this, it must be verified that the optimal technical planning is feasible thanks to an operational planning. Indeed, some of the constraints of the maintenance yard cannot be implemented in the technical planning mathematical model. This highlights the fact that a new mathematical model, that would take the optimal technical planning as an input, should be created. Indeed, with both an optimal technical planning and an optimal operational planning found, the total maintenance costs of a company would be significantly improved.

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